Correspondence

Planar Transmission Lines—III*

The general theory of planar transmission lines developed earlier[1] allows one to show that there is only a certain range of dimensions within which the line will be mechanically stable and reproducible. Within this range, very simple approximate formulas for the attenuation and characteristic impedance can be given. The optimal line, with respect to both stability and attenuation, has its central conductors in the form of very narrow strips.

In the earlier communication we showed how one can compute the operating parameters (characteristic impedance, Z_0, and attenuation, α) of transmission lines made of thin conducting strips supported by dielectric slabs, as shown in cross section in Fig. 1. The study resulted in a number of more or less complex formulas in which the physical relationships were much obscured, and the question was subsequently raised whether there was at least some criterion which can be set up for a satisfactory line, accompanied by practical simplifications of the basic formulas. It has become clear what the main criterion is. This note will describe it and exhibit simple approximations to α and Z_0 for lines which satisfy the criterion.

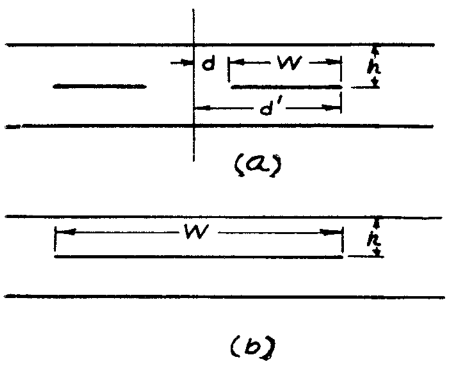

Fig. 1—Cross sections of two planar transmission lines.

If one examines Fig. 4 of I and the associated formulas, it becomes clear that the line parameters depend very sensitively on the quantity k, defined by

$$k = \frac{\sinh d/b}{\sinh d'/b} \quad (b = 2h/\pi) \qquad (1)$$

when k is in the neighborhood of 0 or 1. In such a case, a very small change in any of the dimensions of the line may result in totally different operating characteristics. Such a line is unsuitable for practical use; for stability we must require that

$$0.1 < k^2 < 0.9, \qquad (2)$$

or some similar restrictive condition.

Suppose that the line has $d \geqq b$. Then it follows from (1) that to reasonable accuracy[2]

$$k^2 \approx e^{-2w/b} \qquad (3)$$

and putting this into (2) results essentially in

$$0.03 < w/h < 0.75. \qquad (4)$$

Thus the primary requirement on the strips is that they be relatively narrow. If the dielectric is rather thin, then the strips must be narrow ribbons, and can of course be laid at any distance apart. It seems likely, however, that convenience of manufacture would require the inequality $d \geqq b$ to be satisfied by quite a comfortable margin, and so we shall assume it to be true throughout.

If k^2 is in the middle range (2), then the complicated function $f(k^2)$ may for practical purposes be replaced by a straight line (cf. *I*, Fig. 4). One readily finds that, to within a few per cent,

$$K'/K \approx 1 - 0.91(k^2 - \tfrac{1}{2})$$

where

$$0.1 < k^2 < 0.9, \qquad (5)$$

so that the characteristic impedance becomes

$$Z_0 \approx \frac{260}{\sqrt{\kappa}(1 - \frac{5}{8}k^2)} \text{ohms} \qquad (6)$$

where κ is the dielectric constant of the insulating sheets.

The attenuation in the line is given by

$$\alpha = \frac{1}{8}\eta \frac{\epsilon}{\mu} \frac{Z_0}{K^2} J, \quad \eta = \sqrt{(\pi f \mu/\sigma_c)} \quad (7)$$

for the double-strip line and four times as much for the single-strip one, where f is the frequency, σ_c is the conductivity of the conductors, and J is an integral over the boundaries of the conductors which gives the dissipation. If one calculates J under the above restrictions from (26) and (28) or (29) of *I*, one finds that the integrals simplify to

$$J_1 \approx \frac{4w}{b^2k'^2}, \quad J_2 \approx \frac{4}{b^2k'^2}\left(\ln \frac{2w}{t} - \frac{w}{2b}\right) \quad (8)$$

where t is the thickness of the inner strips. Thus, for the two types we have

$$J = J_1 + 2J_2 \approx \frac{8}{bk'^2} \ln \frac{2w}{t} \quad \text{(double strip)} \quad (9a)$$

and

$$J = J_1 + J_2 \approx \frac{4}{bk'^2} \left(\ln \frac{2w}{t} + \frac{w}{2b}\right) \quad \text{(single strip)}. \quad (9b)$$

To continue, one has to approximate the value of k'^2K^2 in the range of values we are considering. To do so we can use the highly convergent series[3]

$$\sqrt{(2k'K/\pi)} = 1 - 2q + 2q^4 + 2q^9 - \cdots$$

where q (cf. I) is given for our purposes by $k^2/16$. Thus,

$$k'^2K^2 \approx \tfrac{1}{2}\pi(1 - \tfrac{1}{8}k^2)^4. \qquad (10)$$

The figure for the attenuation thus involves the factor

$$(1 - \tfrac{1}{8}k^2)^{-4}(1 - \tfrac{5}{8}k^2)^{-1} \approx 1 + k^2 + \tfrac{5}{6}k^4 + \cdots$$

where the coefficients are only approximate. Since there is usually no necessity for an exact evaluation (especially since we have omitted the dissipation in the dielectric material), we shall write the attenuation in the two cases as

$$\alpha \approx 9\cdot 3\cdot 10^{-4}\eta\sqrt{\kappa}(1 + k^2) \ln (2w/t) \quad \text{(double strip)} \quad (11)$$

and

$$\alpha \approx 1\cdot 9\cdot 10^{-3}\eta\sqrt{\kappa}(1+k^2)[\ln (2w/t+w/2b] \quad \text{(single strip)}. \quad (12)$$

For the stable lines, these replace the approximations given in *I*.

Both formulas state that the attenuation will be decreased by decreasing w (down to rather small values). Thus, criteria both of stability and of attenuation demand that the lines be made with w very small (*i.e.* of the order of $\frac{1}{2}h$ or less) which may well conflict with considerations of technical convenience if the strips are made of foil. On the other hand, if the current to be carried is not large, it is indicated by these results that the best lines are exactly those most easily constructed by the use of printing techniques.

I would like to thank Mr. Norton Cushman for his comments on this subject.

D. Park
Fulbright Lecturer
University of Ceylon
Colombo, Ceylon

* Supported by the Sprague Electric Co., North Adams, Mass.

[1] D. Park, "Planar transmission lines," Trans. IRE, vol. MTT-3, pp. 8–12; April, 1955. This will be referred to as I. It contains the following nontrivial errors: p. 11, end: J_2 is the dissipation from both sides of one strip; p. 12, line 5: $e^{-2d/b}$ should be $e^{-2d'/b}$; p. 12, 2 lines after (34): the voltage across the two-strip line is $2K$, not K. None of the subsequent conclusions are altered. One should note also that there are two quantities denoted by w. Both are fully explained; w has one meaning prior to (17) and the other thereafter.

[2] If $d = b$, the error is about 10 per cent, decreasing rapidly as d increases.

[3] E. T. Whittaker and G. N. Watson, "Modern Analysis," Cambridge University Press, Cambridge, Eng., 4th ed., p 479; 1927.

Phase Shift and Attenuation in a Transmission Line

Circular transmission line charts, *i.e.* Carter (Z-θ) and Smith (R-X) charts, can be used to determine the phase shift and attenuation in a uniform transmission line, or waveguide, which may be lossy and which is not terminated in a matched load. The procedure discussed here is based on the simple properties of these charts. The procedure can be extended to symmetrical four-terminal networks.

A uniform transmission line may be described by its characteristic impedance Z_0, propagation constant $\gamma = \alpha + j\beta$, and length d. The units for α and β are nepers and radians per unit length, respectively. When the transmission line is terminated in a matched load, the efficiency of power transfer is

$e^{-2\alpha d}$ and the phase shift is βd. These simple expressions for the efficiency and phase shift are not valid when the transmission line is not terminated in a matched load.

The load voltage E_L, load current I_L, input voltage E_A, and input current I_A are given by the equations

$$E_L = E_L^+(1 + K_L), \quad (1)$$
$$I_L = (E_L^+/Z_0)(1 - K_L), \quad (2)$$
$$E_A = E_L^+(e^{\gamma d} + K_L e^{-\gamma d}) = E_L^+ e^{\gamma d}(1 + K_A), \quad (3)$$
$$I_A = (E_L^+ e^{\gamma d}/Z_0)(1 - K_A); \quad (4)$$

where E_L^+ is the component of voltage at the load associated with the wave traveling toward the load, the voltage reflection coefficient K_L at the load is

$$K_L = \frac{(Z_L/Z_0) - 1}{(Z_L/Z_0) + 1}, \quad (5)$$

and the voltage reflection coefficient K_A at the input is

$$K_A = K_L e^{-2\gamma d}. \quad (6)$$

It follows from (1)–(4) that

$$\frac{E_L}{E_A} = \frac{1 + K_L}{1 + K_A} e^{-\gamma d} \quad (7)$$

and

$$\frac{I_L}{I_A} = \frac{1 - K_L}{1 - K_A} e^{-\gamma d}. \quad (8)$$

The input impedance Z_A is

$$Z_A = \frac{E_A}{I_A} = Z_0 \frac{1 + K_A}{1 - K_A}. \quad (9)$$

The efficiency η of power transfer is

$$\eta = \frac{E_L I_L \cos \theta_L}{E_A I_A \cos \theta_A} = \frac{I_L^2 R_L}{I_A^2 R_A} = \frac{|1 + K_L|\,|1 - K_L| \cos \theta_L}{|1 + K_A|\,|1 - K_A| \cos \theta_A} e^{-2\alpha d} = \frac{|1 - K_L|^2 R_L}{|1 - K_A|^2 R_A} e^{-2\alpha d}, \quad (10)$$

where θ_L and θ_A are the angles of Z_L, and Z_A, and R_L and R_A are the resistance components of Z_L and Z_A, respectively. The efficiency can be converted into attenuation.

Circular transmission line charts provide a convenient graphical method for evaluating (7)–(10). First the normalized load impedance $Z_L' = Z_L/Z_0$ is plotted on a transmission line chart as shown in Fig. 1. This point is rotated about the center C of the chart through the angle $2\beta d$, which corresponds to d/λ, where λ is the wavelength in the transmission line. The new point is designated Z_A''. The distance $\overline{CZ_A''}$ is measured with any convenient scale and is multiplied by $e^{-2\alpha d}$. The point Z_A' is located on the line CZ_A'' at the distance $\overline{CZ_A''}e^{-2\alpha d}$ from C. (If the transmission line is lossless, $Z_A' = Z_A''$.) Lines are drawn through the points Z_L' and Z_A' as shown in Fig. 1.

Now (7)–(10) can be written

$$\frac{E_L}{E_A} = \frac{\overline{OZ_L'}}{\overline{OZ_A'}} e^{-\alpha d}, \quad (7')$$

$$\text{angle of } \frac{E_L}{E_A} = -\left(\frac{d}{\lambda} - \frac{a}{\lambda}\right) 360° \quad (7'')$$

$$\frac{I_L}{I_A} = \frac{\overline{BZ_L'}}{\overline{BZ_A'}} e^{-\alpha d} \quad (8')$$

$$\text{angle of } \frac{I_L}{I_A} = -\left(\frac{d}{\lambda} + \frac{b}{\lambda}\right) 360°, \quad (8'')$$

$$Z_A = Z_0 Z_A', \quad (9')$$

and

$$\eta = \frac{(\overline{OZ_L'})(\overline{BZ_L'}) \cos \theta_L}{(\overline{OZ_A'})(\overline{BZ_A'}) \cos \theta_A} e^{-2\alpha d} = \frac{(\overline{BZ_L'})^2 R_L}{(\overline{BZ_A'})^2 R_A} e^{-2\alpha d}. \quad (10')$$

The distances $\overline{OZ_L'}$, $\overline{OZ_A'}$, $\overline{BZ_L'}$, and $\overline{BZ_A'}$ are measured with any convenient scale. In (7''), the sign of a/λ is positive if the point Z_L' is above the line OZ_A'; otherwise it is negative. In (8''), the sign of b/λ is positive if the point Z_L' is above the line BZ_A'; otherwise it is negative.

If Z_0 and γd are not known, their values can be determined experimentally. Let Z_{sc} and Z_{oc} denote the values of Z_A when $Z_L = 0$ and ∞, respectively. Since

$$Z_{sc} = Z_0 \frac{1 - e^{-2\gamma d}}{1 + e^{-2\gamma d}} \quad (11)$$

and

$$Z_{oc} = Z_0 \frac{1 + e^{-2\gamma d}}{1 - e^{-2\gamma d}}, \quad (12)$$

it follows that

$$Z_0 = \sqrt{Z_{sc} Z_{oc}} \quad (13)$$

and

$$\sqrt{\frac{Z_{sc}}{Z_{oc}}} = \frac{1 - e^{-2\gamma d}}{1 + e^{-2\gamma d}}. \quad (14)$$

For convenience in notation, let $D = \sqrt{Z_{sc}/Z_{oc}}$. Now

$$e^{-2\gamma d} = \frac{1 - D}{1 + D}. \quad (15)$$

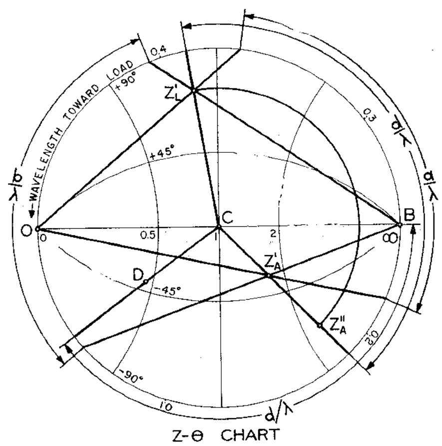

Fig. 1—Graphical construction.

When the point D is plotted on a transmission line chart as shown in Fig. 1, d/λ is measured as indicated and

$$e^{-2\alpha d} = \frac{\overline{CD}}{\overline{OC}}. \quad (16)$$

The values of the parameters for the construction shown in Fig. 1 are $Z_0 = 100/\underline{-10°}$, $Z_L = 85/\underline{65°}$, $d = 0.2\lambda$, and $e^{-\alpha d} = 0.707$. The results are $E_L/E_A = 0.63/\underline{-19°}$, $I_L/I_A = 1.24/\underline{-126°}$, $Z_A = 168/\underline{-42°}$, and $\eta = 0.45$.

H. F. Mathis
Goodyear Aircraft Corp.
Akron, Ohio

A Novel Technique for Making Precision Waveguide Twists

In a recent issue of the IRE Transactions on Microwave Theory and Techniques, Wheeler and Schwiebert[1] described step-twist waveguide components having good performance over a full waveguide band. These step-twist components are composed of a few short sections of straight rectangular waveguide, twisted about their common axis at their junction faces. Typically the length of each short section of the step-twist is the order of ⅛ to ¼ of a guide wavelength, while the twist angle between sections may be as much as 30 degrees.

The technique[2] described here for constructing precision waveguide twists makes use of a large number of very short adjoining sections of waveguide. Hence it can be considered as the limiting case of Wheeler's and Schwiebert's step-twist technique. The twists constructed of these short sections of waveguide can easily be made to assume very complicated shapes that would be practically impossible to construct in any other manner. Furthermore, their electrical properties, such as vswr, attenuation, and power-handling capacity, are essentially the same as those of straight sections of waveguide.

The short lengths of waveguide used in these twists are stampings formed by a precision punch and die, and consequently can be inexpensively mass produced. Fig. 1 shows two typical stampings of 0.005-inch brass sheet suitable for X-band components.

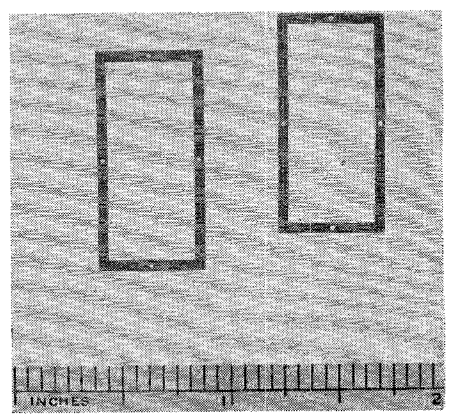

Fig. 1—Stampings used for constructing X-band precision twists.

One convenient method for aligning the stampings is to string them together on wires by means of the small holes shown in the figure. When the stampings are strung on the wires, the complete stack can then be twisted and one end translated with respect to the other in any desired manner, keeping the

[1] H. A. Wheeler and H. Schwiebert, "Step-twist waveguide components," Trans. IRE, vol. MTT-3, pp. 45–51; October, 1955.

[2] This construction technique was evolved by R. R. McPherson of Stanford Res. Inst. when called upon by the authors to make the multiple waveguide twist section described later in this letter.